A high-precision study of polarized proton scattering to low-lying states in 11B

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2. Relevant theory

The following sections give an overview of the theoretical foundations needed for the analysis performed in this work. The first section deals with the distorted-wave description of nucleon-nucleus scattering and the input required to perform numerical calculations on the process. Section 2.2 gives a short introduction to the shell-model methods applied to obtain nuclear-structure information and section 2.3 presents the formalism used to describe polarization-transfer experiments.

2.1 Scattering formalism

The following subsections present the formalism involved in the microscopic description of nucleon-nucleus scattering based on different forms of the distorted-wave approximation. In this approximation the incoming and outgoing particles are described by plane and spherical waves which get distorted by some average nuclear potential usually tuned to reproduce elastic scattering. Inelastic transitions are mediated by an effective interaction between the projectile and the individual nucleons of the nucleus. The cross section for a particular nuclear excitation then depends on the effective interaction and on nuclear-structure information which is embedded in the theory via so-called one-body transition densities.
An in-depth description of the formalism can be found in the books of Jackson [26] and Glendenning [27]. The theoretical formulation of nucleon-nucleus scattering in the distorted-wave approximation has been cast, among others, into the computer code DWBA98 by Raynal [28]. Together with the above mentioned ingredients this code has been used to perform the necessary calculations in this work.

2.1.1 Lippmann-Schwinger equation

The scattering of a projectile from a potential $V(\vec{r})$ is described by the time-independent Schrödinger equation

$$(E - H_0)\psi(\vec{k}, \vec{r}) = V(\vec{r})\psi(\vec{k}, \vec{r}) \ , \tag{2.1}$$

where $\vec{r}$ is the spatial vector, $\vec{k}$ the wave vector and H_0 the kinetic-energy operator. E is the energy in the center-of-mass system, given by

$$E = \frac{\hbar^2}{2\mu}k^2 \tag{2.2}$$

with μ being the reduced mass of the system.
A solution of equation 2.1 has to be found with the boundary condition that the total wave function must have the asymptotic behavior of an incoming plane wave and an outgoing spherical wave

$$\psi(\vec{k},\vec{r}) \overset{r\to\infty}{\longrightarrow} \exp(i\vec{k}\cdot\vec{r}) + f(\theta,\phi)\frac{\exp(ikr)}{r} \ , \tag{2.3}$$

with $k = |\vec{k}|$ and $r = |\vec{r}|$. $f(\theta,\phi)$ is called the scattering amplitude and is connected to the differential cross section by [1]

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = |f(\theta,\phi)|^2 \ . \tag{2.4}$$

By expanding $\psi(\vec{k},\vec{r})$ into plane-wave states, which are solutions of

$$(E - H_0)\phi(\vec{k}\,',\vec{r}) = 0 = (E - E')\phi(\vec{k}\,',\vec{r}) \ . \tag{2.5}$$

and using the orthogonality and closure relations for these states one gets

$$\psi(\vec{k},\vec{r}) = \int G_0(\vec{r},\vec{r}\,')V(\vec{r}\,')\psi(\vec{k},\vec{r}\,')d\vec{r}\,' \ , \tag{2.6}$$

with

$$G_0(\vec{r},\vec{r}\,') = (2\pi)^{-3}\int \frac{\phi(\vec{k}\,',\vec{r})\phi^*(\vec{k}\,',\vec{r}\,')}{E - E'}d\vec{k}\,' \ . \tag{2.7}$$

Adding to 2.6 a solution of 2.5 one obtains the general solution of equation 2.1

$$\psi(\vec{k},\vec{r}) = \phi(\vec{k},\vec{r}) + \int G_0(\vec{r},\vec{r}\,')V(\vec{r}\,')\psi(\vec{k},\vec{r}\,')d\vec{r}\,' \ . \tag{2.8}$$

A disadvantage of equation 2.7 is that it has poles at $k' = \pm k$. To be able to perform the integration, an infinitesimal quantity $\pm i\epsilon$ is added to the denominator, which is taken to zero after the integration has been performed. This results in

$$G_0^{\pm}(\vec{r},\vec{r}\,') = -\frac{\mu}{2\pi\hbar^2}\frac{\exp(\pm ik|\vec{r}-\vec{r}\,'|)}{|\vec{r}-\vec{r}\,'|} \ , \tag{2.9}$$

[1] In case of inelastic scattering where the magnitude of the incoming and outgoing momenta is different, one gets an additional factor k_f/k_i in the cross section.

which, if one assumes that the interaction $V(\vec{r}\,')$ falls off rapidly with r', gives the following asymptotic behavior of equation 2.8

$$\psi^{\pm}(\vec{k},\vec{r}) \stackrel{r\to\infty}{\longrightarrow} \exp(i\vec{k}\cdot\vec{r}) - \frac{\mu}{2\pi\hbar^2}\frac{\exp(\pm ikr)}{r}\int \exp(\mp i\vec{k}\,'\cdot\vec{r}\,')V(\vec{r}\,')\psi^{\pm}(\vec{k},\vec{r}\,')d\vec{r}\,' \,. \tag{2.10}$$

Equation 2.10 does indeed fulfill the required boundary conditions. Comparing with 2.3 yields for the scattering amplitude

$$f(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\int \exp(-i\vec{k}\,'\cdot\vec{r}\,')V(\vec{r}\,')\psi^{+}(\vec{k},\vec{r}\,')d\vec{r}\,' \,. \tag{2.11}$$

For the further discussion equation 2.8 has to be expressed in an operator formalism. For that purpose, the Schrödinger equation is multiplied by $(E-H_0)^{-1}$

$$\psi(\vec{k},\vec{r}) = (E-H_0)^{-1}V(\vec{r})\psi(\vec{k},\vec{r}) \,, \tag{2.12}$$

to which a solution of 2.5 is added and the divergence at $E=E'$ is removed by inserting an infinitesimal quantity $\pm i\epsilon$ obtaining

$$\psi^{\pm}(\vec{k},\vec{r}) = \phi(\vec{k},\vec{r}) + (E-H_0\pm i\epsilon)^{-1}V(\vec{r})\psi^{\pm}(\vec{k},\vec{r}) \,, \tag{2.13}$$

which is a solution of the Schrödinger equation in the limit $\epsilon\to 0$. Comparing with 2.8 one can identify $G_0^{\pm}(\vec{r},\vec{r}\,')$ with the Green's function for the operator

$$G_0^{\pm} = \frac{1}{E-H_0\pm i\epsilon} \tag{2.14}$$

and thus rewriting equation 2.13 in operator form one gets

$$|\psi^{\pm}\rangle = |\phi\rangle + G_0^{\pm}V|\psi^{\pm}\rangle \tag{2.15}$$

the so-called *Lippmann-Schwinger equation.* Defining for the total Hamiltonian $H=H_0+V$

$$G^{\pm} = \frac{1}{E-H\pm i\epsilon} \,, \tag{2.16}$$

one finds, using some operator algebra, the relations

$$G^{\pm} = G_0^{\pm} + G_0^{\pm}VG^{\pm} \tag{2.17}$$

and

$$G_0^{\pm} = G^{\pm} - G^{\pm}VG_0^{\pm} \,. \tag{2.18}$$

Substituting the latter one into 2.15 gives

$$|\psi^{\pm}\rangle = |\phi\rangle + G^{\pm}V|\phi\rangle \tag{2.19}$$

which is a formal solution of the Lippmann-Schwinger equation. Because $G^{\pm}$ contains the full Hamiltonian, equation 2.19 cannot be used directly to calculate the scattering wave function $\psi^{\pm}(\vec{k},\vec{r})$, but is useful as the starting point of various approximations.

2.1.2 Distorted-wave approximations

Equation 2.17 has an iterative solution given by

$$G^{\pm} = G_0^{\pm} + G_0^{\pm} V G_0^{\pm} + \cdots = \sum_{n=0}^{\infty} (G_0^{\pm} V)^n G_0^{\pm} \ , \tag{2.20}$$

which, when inserted into equation 2.19, gives the so-called *Born expansion* of the scattering wave function

$$|\psi^{\pm}\rangle = |\phi\rangle + \sum_{n=0}^{\infty} (G_0^{\pm} V)^n G_0^{\pm} V |\phi\rangle \ . \tag{2.21}$$

The physical interpretation of this expansion is that the incoming plane wave is undergoing successive scatterings from the potential V with $G_0^{\pm}$ acting as a propagator.
One can define transition operators $T^{\pm}$ by

$$T^{\pm} = V + V G^{\pm} V = V + V G_0^{\pm} T^{\pm} \ , \tag{2.22}$$

with the transition matrix element

$$T_{fi} = \langle \phi | T^{+} | \phi \rangle = \langle \phi | V | \psi^{+} \rangle \ , \tag{2.23}$$

where i and f are used to label the initial and final scattering wave functions, respectively. The matrix element T_{fi} can be related to the scattering amplitude by comparing with 2.11, which leads to

$$f(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2} T_{fi} \ . \tag{2.24}$$

For use in so-called *distorted-wave approximations* the potential V is split up into two terms,

$$V = U + W \ , \tag{2.25}$$

such that the scattering problem with the potential U alone has a known solution χ which fulfills the equation

$$(H_0 + U)\chi^{\pm} = E\chi^{\pm} \ . \tag{2.26}$$

Defining

$$G_1^{\pm} = \frac{1}{E - H_0 - U \pm i\epsilon} \; , \; G^{\pm} = \frac{1}{E - H_0 - U - W \pm i\epsilon} \; , \tag{2.27}$$

one can express $|\psi^{\pm}\rangle$ in terms of the wave functions $|\chi^{\pm}\rangle$ by

$$|\psi^{\pm}\rangle = |\chi^{\pm}\rangle + G_1^{\pm} W |\psi^{\pm}\rangle \; . \tag{2.28}$$

Performing some algebra using $G_0^- = (G_0^+)^{\dagger}$, the transition matrix element from equation 2.23 can then be written as

$$T_{fi} = \langle \phi | U | \chi^+ \rangle + \langle \chi^- | W | \psi^+ \rangle \tag{2.29}$$

and by inserting the iterative solution of equation 2.28 one gets

$$T_{fi} = \langle \phi | U | \chi^+ \rangle + \sum_{n=0}^{\infty} \langle \chi^- | W (G_1^+ W)^n | \chi^+ \rangle \; . \tag{2.30}$$

Up to now the scattering process was considered as the interaction of a projectile with some potential V. To describe inelastic scattering connecting different states of the target nucleus, the operators $G_1^{\pm}$ and $G^{\pm}$ are modified to contain the nuclear Hamiltonian H_N,

$$G_1^{\pm} = \frac{1}{E - H_0 - U - H_N \pm i\epsilon} \; , \; G^{\pm} = \frac{1}{E - H_0 - U - W - H_N \pm i\epsilon} \tag{2.31}$$

and U is chosen to represent elastic scattering from the target, transforming the incoming plane-wave states into distorted waves. Because in that case U is not connecting different target states the first term in 2.30 drops out. The potential U does, however, have an effect on T_{fi} by determining the form of the distorted waves $\chi^{\pm}$. The transition matrix element then becomes

$$\begin{aligned} T_{fi} &= \langle \chi_f^- \Phi_f | W | \psi_i^+ \Phi_i \rangle \\ &= \sum_{n=0}^{\infty} \langle \chi_f^- \Phi_f | W (G_1^+ W)^n | \chi_i^+ \Phi_i \rangle \; , \end{aligned} \tag{2.32}$$

where Φ_i and Φ_f represent the initial and final nuclear wave functions, respectively.

Terminating 2.32 after the first term of the summation gives the *distorted-wave Born approximation* (DWBA) which physically means that multiple scattering reactions where the projectile interacts more than once with the

potential are neglected. Using 2.24 the differential cross section in DWBA takes the form

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}|T_{fi}|^2 = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}|\langle\chi_f^-\Phi_f|W|\chi_i^+\Phi_i\rangle|^2 \ . \quad (2.33)$$

A slightly different approximation can be obtained by writing the transition potential as

$$W = \sum_j W_j \quad (2.34)$$

where W_j is the potential caused by the j^{th} nucleon in the target nucleus. A transition operator t_j can then be defined which describes the scattering of the projectile from this target nucleon

$$t_j = W_j + W_j G_1^+ t_j \ . \quad (2.35)$$

Using 2.34 and 2.35 one can derive the following form of the transition matrix element [27]

$$\begin{aligned} T_{fi} &= \langle\chi_f^-\Phi_f|\sum_j W_j|\psi_i^+\Phi_i\rangle \\ &= \langle\chi_f^-\Phi_f|\sum_j t_j|\chi_i^+\Phi_i\rangle + \langle\chi_f^-\Phi_f|\sum_{j,\,k\neq j} t_j G_1^+ t_k|\chi_i^+\Phi_i\rangle + \ldots \ . \end{aligned} \quad (2.36)$$

The transition operator t_j sums the interaction with the j^{th} nucleon to all orders so that the first term in 2.36 gives a complete description of the scattering from any single target nucleon. The second term in the series describes two-step processes where the projectile interacts successively with any two bound nucleons. Terminating the series expansion after the first term gives the *single-scattering distorted-wave approximation* which is believed to be a good approximation in the intermediate energy regime (at projectile energies above $\sim$ 100 MeV) [29].

2.1.3 Effective interactions

Two approaches are taken to describe the interaction of the projectile with the target nucleons for use in actual calculations. In the *distorted-wave impulse approximation* (DWIA) which builds upon the *single-scattering approximation* presented in the last section, the operator t_j is identified with

the operator for the scattering of free particles

$$t_j = W_j + W_j \frac{1}{E - H_0 + i\epsilon} t_j \tag{2.37}$$

thereby neglecting all effects of the nuclear medium on the interaction. This approximation is believed to become valid at projectile energies exceeding 150 MeV [5]. In some cases, however, medium effects due to Pauli blocking are known to play a role also at higher energies. Parameterizations of the free nucleon-nucleon t-matrix for use in actual calculations have among others been developed by Franey and Love [5, 30].
A way to include medium effects in the description of nucleon-nucleon scattering is given by Brueckner's $\mathcal{G}$-matrix which is defined by the Bethe-Goldstone equation

$$\mathcal{G}_j = W_j + W_j \frac{Q}{E - H_1 + i\epsilon} \mathcal{G}_j \tag{2.38}$$

where Q is the Pauli exclusion operator blocking intermediate states which are forbidden due to the Pauli principle and H_1 is a Hamiltonian which gives the energy of the two nucleons moving in the mean field of the nucleus.
Several parameterizations of effective interactions based on the $\mathcal{G}$-matrix are available for use in distorted-wave calculations. These are either developed from the Paris-NN potential [31], as is the case for the parameterizations of Von Geramb [32] and Dortmans and Amos [18], or from the Bonn-NN potential [33] which has been used by Nakayama and Love [34].
The different parameterizations represent the nonlocal $\mathcal{G}$- or t-matrix by a local operator having the form

$$v(\vec{r}, \vec{p}) = v_C(r) + v_{LS}(r) \vec{L} \cdot \vec{S} + v_T(r) S_{12} \tag{2.39}$$

with

$$\begin{gathered} \vec{L} = \vec{r} \times \vec{p}, \quad \vec{r} = \vec{r}_1 - \vec{r}_2, \quad \vec{p} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2) \ , \\ \vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \ , \\ S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad \hat{r} = \vec{r}/|\vec{r}| \ . \end{gathered} \tag{2.40}$$

The central part v_C of the operator has both spin- and isospin-dependent components and the spin-orbit and tensor parts v_{LS} and v_T have isoscalar and isospin-dependent terms,

$$\begin{aligned} v_C(r) &= v_0(r) + v_\sigma(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + v_\tau(r) \vec{\tau}_1 \cdot \vec{\tau}_2 + v_{\sigma\tau}(r) (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) \ , \\ v_T(r) &= v_T^0(r) + v_T^1(r) \vec{\tau}_1 \cdot \vec{\tau}_2 \ , \\ v_{LS}(r) &= v_{LS}^0(r) + v_{LS}^1(r) \vec{\tau}_1 \cdot \vec{\tau}_2 \ . \end{aligned} \tag{2.41}$$

The potential functions $v_\alpha(r)$ are represented as sums of Yukawa potentials of various ranges r_n for the central and spin-orbit components

$$v_\alpha(r) = \sum_n V_n^\alpha \frac{\exp(-r/r_n)}{r/r_n} \tag{2.42}$$

and as sums of Yukawa functions multiplied by r^2 for the tensor components. The strengths and ranges of each component are adjusted so that the momentum-space representation of the operator gives the best possible approximation to the underlying t- or $\mathcal{G}$-matrix.
An interaction of the form 2.39 has to be evaluated with properly antisymmetrized wave functions of the projectile and the bound nucleons. Equivalently one can use unsymmetrized wave functions and replace the interaction $v(\vec{r},\vec{p})$ by

$$\hat{v}(\vec{r},\vec{p}) = v(\vec{r},\vec{p})(1 - P_{12}) \ , \tag{2.43}$$

where $P_{12} = P^x P^\sigma P^\tau$ is the operator which exchanges all coordinates of the particles 1 and 2. It is however possible, as is derived in the book of Glendenning [27], to replace the exchange contribution $-v(\vec{r},\vec{p})P_{12}$ by an effective operator such that

$$\hat{v}(\vec{r},\vec{p}) \simeq \bar{v}(\vec{r},\vec{p}) = v(\vec{r},\vec{p}) + \bar{v}^E(\vec{r},\vec{p}) \ . \tag{2.44}$$

The direct matrix element of the effective operator $\bar{v}^E(\vec{r},\vec{p})$ approximates the exchange parts, which means that only direct matrix elements have to be computed in the calculation of the transition amplitude T_{fi}.

2.1.4 The optical-model potential

The distorted waves $\chi^\pm$ introduced in section 2.1.2 are obtained from solving the Schrödinger equation 2.1 with some average potential U which is created by the nucleus. U is usually tuned to represent elastic scattering leading to a phenomenological optical-model potential. The conventional form of this potential contains real and imaginary parts of central and spin-orbit type and is parameterized as [35]

$$\begin{aligned} U(r) \ &= \ V_C(r) - V_0 f(x_0) - i[W f(x_W) - 4W_D \frac{d}{dx_D} f(x_D)] \\ &\quad + 2(V_{so} + iW_{so}) \frac{1}{r}\frac{d}{dr} f(x_{so}) \vec{L}\cdot\vec{S} \end{aligned} \tag{2.45}$$

where

$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{r}, & \text{for } r \geq R_C \\ \frac{Z_1 Z_2 e^2}{2R_C}(3 - \frac{r^2}{R_C^2}), & \text{for } r \leq R_C \end{cases} ; \quad (2.46)$$

$$R_C = r_C A^{1/3} ; \quad (2.47)$$

$$f(x_i) = (1 + \exp(x_i))^{-1} ; \quad (2.48)$$

$$x_i = (r - r_i A^{1/3})/a_i . \quad (2.49)$$

$V_C(r)$ is the Coulomb potential of a uniformly charged sphere of radius R_C and the functions $f(x_i)$ are Woods-Saxon form factors with radius r_i and diffuseness parameter a_i. In the phenomenological approach the potential depths V_0, W, W_D, V_{so} and W_{so} and the geometrical parameters r_i and a_i are adjusted to give the best fit to elastic cross-section and analyzing-power data.
A different way to obtain an optical potential is to fold the ground-state density of the nucleus with the effective NN interaction used in the distorted-wave calculations. This has the advantage of circumventing the well-known ambiguities [17] involved in the phenomenological determination of optical-model parameters. The methods to perform the folding and proper anti-symmetrization of the projectile and target nucleons are contained in the code DWBA98 [28].

2.1.5 One-body transition densities

When applying the approximations described so far, the scattered particle plays the role of a one-body operator acting on the target nucleons. This makes it possible to write any inelastic nuclear transition as a superposition of single-particle transitions taking target nucleons from an orbit j to another orbit j' with amplitudes $S_J^{j'j}(J_f, J_i)$.
The separation of nuclear-structure and interaction contributions to the scattering amplitude is achieved by performing a multipole expansion of the effective interaction in momentum space [27], giving

$$T_{fi} = \frac{2}{\pi} \sum_{\alpha\beta JM} (-1)^M \int v_{\beta\alpha}(q, Q) \langle \chi_f | T_{LSJ}^{-M(\beta)} | \chi_i \rangle \langle \Phi_{J_f}^{M_f} | \sum_{j=1}^{A} T_{LSJ}^{M(\alpha)}(j) | \Phi_{J_i}^{M_i} \rangle \, q^2 dq \quad (2.50)$$

with

$$\vec{Q} = \vec{k}_i + \vec{k}_f, \ \vec{q} = \vec{k}_i - \vec{k}_f . \quad (2.51)$$

In equation 2.50, β and α represent the quantum numbers of the projectile and target, respectively, $v_{\beta\alpha}(q,Q)$ are Bessel transformations of the various parts of the effective interaction and $T_{LSJ}^{M(\alpha,\beta)}$ are spherical tensor operators of the form

$$T_{LSJ}^{M} = \begin{cases} Y_L^M(\hat{r})\,\delta_{LJ}\,, & \text{for } S=0 \\ [Y_L(\hat{r})\vec{\sigma}]_J^M\,, & \text{for } S=1 \end{cases} \tag{2.52}$$

where the square brackets denote vector coupling and $Y_L^M(\hat{r})$ are spherical harmonics. Using the Wigner-Eckart theorem the nuclear matrix element can be written as

$$\langle \Phi_{J_f}^{M_f} | \sum_{j=1}^{A} T_{LSJ}^{M}(j) | \Phi_{J_i}^{M_i} \rangle = (-1)^{J_f - M_f} \begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix} \langle \Phi_{J_f} || \sum_{j=1}^{A} T_{LSJ}(j) || \Phi_{J_i} \rangle \,. \tag{2.53}$$

The reduced matrix element in equation 2.53 can be expressed as a sum of single-particle matrix elements $\langle j' || T_{LSJ} || j \rangle$ weighted with so-called one-body transition densities $S_J^{j'j}(J_f, J_i)$

$$\langle \Phi_{J_f} || \sum_{j=1}^{A} T_{LSJ}(j) || \Phi_{J_i} \rangle = \sum_{j'j} S_J^{j'j}(J_f, J_i) \langle j' || T_{LSJ} || j \rangle \,, \tag{2.54}$$

which are defined as

$$S_J^{j'j}(J_f, J_i) = -\frac{1}{\sqrt{2J+1}} \langle \Phi_{J_f} || [\beta_{j'}^{\dagger}, \tilde{\beta}_j]_J || \Phi_{J_i} \rangle \,. \tag{2.55}$$

Here, $\beta_{j'}^{\dagger}$ is a creation operator producing a particle with angular momentum j' and $\tilde{\beta}_j$ an annihilation operator creating a hole state with angular momentum j. One-body transition densities can be calculated from model wave functions of the nuclear states in question using shell-model codes like OXBASH [19]. The calculation of single-particle matrix elements for specific operators is discussed in reference [36].

2.1.6 Electromagnetic transition operators

The tensor operators T_{LSJ}^{M} from section 2.1.5 can be identified with electric and magnetic multipole operators, given by

$$O(EL_{M_L}) = \sum_{k=1}^{A} e_k r_k^L Y_L^{M_L}(\hat{r}_k) \tag{2.56}$$

and

$$O(ML_{M_L}) = \sum_{k=1}^{A} \mu_N \left[g_k^s \vec{s}_k + \frac{2g_k^l}{L+1} \vec{l}_k \right] \vec{\nabla}_k r^L Y_L^{M_L}(\hat{r}_k) \ , \tag{2.57}$$

respectively [36]. In these definitions e_k is the charge of the k^{th} nucleon, g_k^s is the spin g-factor with its free values given by $g_p^s = 5.586$ for protons and $g_n^s = -3.826$ for neutrons and the factor g_k^l is the orbital g-factor which is 1 for protons and 0 for neutrons. The operators follow the usual selection rule for angular momentum coupling, which can be expressed as

$$|J_i - J_f| \leq L \leq J_i + J_f \ . \tag{2.58}$$

Their parities are given by

$$\Pi_{O(ML)} = (-1)^{L+1} \ , \quad \Pi_{O(EL)} = (-1)^L \ , \tag{2.59}$$

leading to the parity selection rule

$$\Pi_f \Pi_{O(\overline{\omega}L)} \Pi_i = +1 \ , \tag{2.60}$$

where Π_i and Π_f are the parities of the initial and final states, respectively. The reduced transition strength $B(\overline{\omega}L; J_i \to J_f)$ is calculated from the matrix element of the electromagnetic transition operator between the wave functions of the initial and final nuclear states

$$B(\overline{\omega}L; J_i \to J_f) = \frac{\langle \Phi_{J_f} || O(\overline{\omega}L) || \Phi_{J_i} \rangle^2}{2J_i + 1} \tag{2.61}$$

and is given in units of $e^2 fm^{2L}$ for $B(EL)$ values and $\mu_N^2 fm^{2L-2}$ for $B(ML)$ values. For conversion into Weisskopf units, which are often used in the literature, see reference [37]. The transition strengths for the excitation and for the de-excitation of a certain state are related by

$$B(\overline{\omega}L; J_i \to J_f) = \frac{2J_f + 1}{2J_i + 1} B(\overline{\omega}L; J_f \to J_i) \ . \tag{2.62}$$

2.2 Shell-model calculations

For the microscopic description of nucleon-nucleus scattering and for the calculation of quantities like electromagnetic transition strengths it is necessary to obtain model wave functions of the nuclear states in question.

The nuclear shell model has provided a successful approach to this problem especially since the rapid growth in computing power made it possible to perform numerical calculations of shell-model wave functions in large, and thus more realistic, configuration spaces [38, 39].
The following sections give a short introduction into the aspects of the nuclear shell model involved in the numerical calculation of model wave functions and the deduction of electromagnetic transition strengths. A complete coverage of shell-model uses in nuclear spectroscopy is contained in the book of Brussaard and Glaudemans [36].

2.2.1 The nuclear Hamiltonian

The shell-model potential felt by a particular nucleon is derived from the interactions with its nearest neighbors. Because of the special character of the interaction, namely being a short-range attractive force with a strong repulsive core, and because of the Pauli principle, which forbids two nucleons to occupy exactly the same quantum-mechanical state, the potential is essentially constant at the center of the nucleus and goes quickly to zero at the nuclear surface. An important representation of this behavior is given by the Woods-Saxon potential

$$V(r) = -\frac{V_0}{1 + \exp((r - R)/a)} \tag{2.63}$$

(see figure 2.1, left panel), which has a radial shape similar to that of the nuclear density distribution. R is the nuclear radius and a the diffuseness of the nuclear surface. To the nuclear potential $V(r)$ one must add the Coulomb potential $V_C(r)$ and a spin-orbit part $V_{so}(r)\vec{l} \cdot \vec{s}$, which is required to give a correct description of the observed shell closures and remove the degeneracy of states with the same orbital angular momentum l (see figure 2.1, right panel). In case of a Woods-Saxon potential the Schrödinger equation has to be solved numerically. Analytic forms of the nuclear wave functions can be obtained by using a harmonic-oscillator potential

$$V(r) = -V_0 + m\omega^2 r^2/2 \tag{2.64}$$

and neglecting the spin-orbit force. The energy levels of the harmonic-oscillator potential are given by

$$E_{nl} = [2(n-1) + l + 3/2]\hbar\omega \tag{2.65}$$

$$b = \sqrt{\hbar/m\omega} \tag{2.66}$$

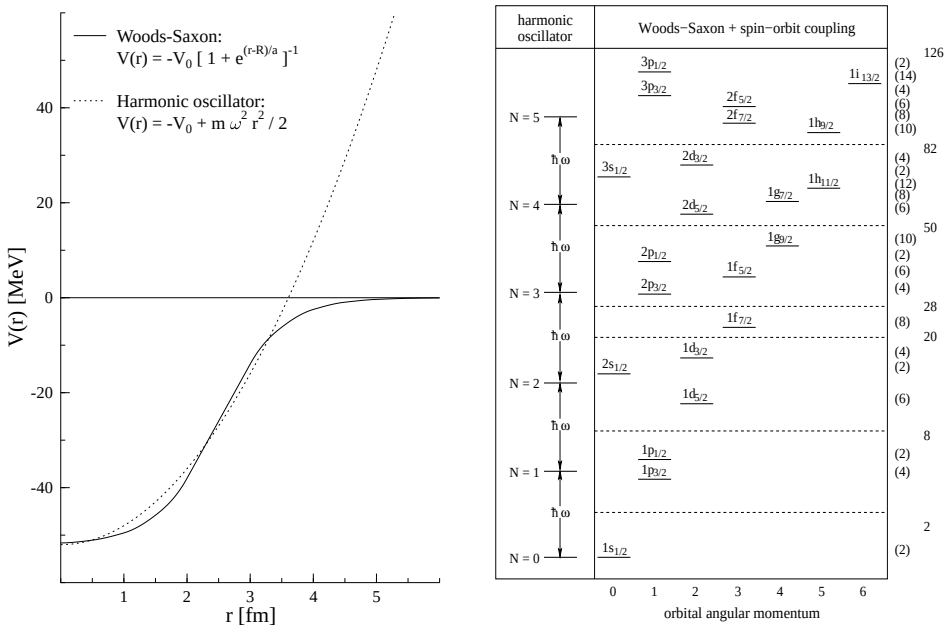

Figure 2.1: Left panel: Woods-Saxon and harmonic-oscillator forms of the shell-model potential. Right panel: Single-particle levels calculated in a harmonic-oscillator potential and in a Woods-Saxon potential with spin-orbit splitting (from reference [36]).

with b being the harmonic-oscillator parameter and m the nucleon mass. The nuclear potential can be written as a sum of the two-body interactions V_{ij} which take place between the nucleons inside the nucleus

$$\begin{aligned} V &= \sum_{i>j}^{A} V_{ij} \\ &= \sum_{i}^{A} V_i + \sum_{i>j}^{A} W_{ij} \ , \end{aligned} \tag{2.67}$$

where V_i is the average (mean-field) potential the i^{th} nucleon feels as a result of the combined force of the rest of the nucleons and W_{ij} represents the residual interactions which cannot be incorporated into the average potential. The average potential V_i causes completely independent motion of the nucleons and leads to pure single-particle wave functions. The residual interactions perturb this picture and cause the nuclear wave function to be-

come a superposition of single-particle states. This situation is known as configuration mixing. Considering equation 2.67 the nuclear Hamiltonian becomes

$$H = \sum_i^A \frac{\hbar^2 p_i^2}{2m} + \sum_i^A V_{ij}(|\vec{r}_i - \vec{r}_j|) \tag{2.68}$$

$$= \sum_i^A \frac{\hbar^2 p_i^2}{2m} + \sum_i^A V_i(\vec{r}_i) + \sum_{i>j}^A W_{ij}(|\vec{r}_i - \vec{r}_j|) \ , \tag{2.69}$$

where $\vec{r}_i$ represents the coordinates of the i^{th} nucleon with respect to an arbitrary origin, $\vec{p}_i$ is its momentum and the residual potential W_{ij} is a function of the distance between the nucleons.

2.2.2 Spurious center-of-mass motion

The conservation of linear momentum requires the nuclear Hamiltonian to be translation invariant, implying that it must rely on relative coordinates only. The use of a single-particle potential with an arbitrary origin of coordinates, as in equation 2.69, introduces components in the nuclear wave function describing spurious collective motion, in which the center-of-mass oscillates around the origin of the potential.
To study the effect of the center-of-mass motion the single-particle potential in 2.69 is replaced by a harmonic-oscillator potential

$$H = \sum_i^A \frac{\hbar^2 p_i^2}{2m} + \sum_i^A \frac{1}{2} m\omega^2 r_i^2 + \sum_{i>j}^A W_{ij}(|\vec{r}_i - \vec{r}_j|) \ . \tag{2.70}$$

Using the *Gartenhaus and Schwartz* transformation [26]

$$\vec{\rho}_i = \vec{r}_i - \vec{R} \ , \qquad \vec{q}_i = \vec{p}_i - \frac{1}{A}\vec{P} \ , \tag{2.71}$$

$$\vec{R} = \frac{1}{A}\sum_i^A \vec{r}_i \ , \qquad \vec{P} = \sum_i^A \vec{p}_i \ , \tag{2.72}$$

the Hamiltonian can then be separated into a part describing the center-of-mass motion and a part depending only on intrinsic coordinates of the nucleus

$$H = \underbrace{\frac{\hbar^2 P^2}{2Am} + \frac{1}{2} Am\omega^2 R^2}_{H_{cm}} + H_{int}(\vec{q}_1 \, , \ldots \, , \vec{q}_{A-1} \, ; \vec{\rho}_1 \, , \ldots \, , \vec{\rho}_{A-1}) \ . \tag{2.73}$$

In that case the wave function becomes

$$\Psi_{\alpha n}(\vec{R}, \vec{\rho}_1 , \ldots , \vec{\rho}_{A-1}) = \phi_\alpha(\vec{R})\Phi_n(\vec{\rho}_1 , \ldots , \vec{\rho}_{A-1}) , \tag{2.74}$$

where ϕ_α is an oscillator function describing the center-of-mass motion in an oscillator potential and Φ_n describes the intrinsic state of the nucleus. The energy spectrum of the center-of-mass motion has the same spacing as that of the single-particle states given by equation 2.66 so that the total energy of the system is given by

$$E_{\alpha n} = (\alpha + 3/2)\hbar\omega + E_n^{int} . \tag{2.75}$$

The internal, or true, energy of the nucleus is

$$E_n^{int} = E_{0n} - 3/2\hbar\omega . \tag{2.76}$$

Spurious states where $\alpha > 0$ can only occur at excitation energies of at least 1 $\hbar\omega$, which is the energy required to raise the center-of-mass into its first excited state. In actual calculations of shell-model wave functions spurious states are usually suppressed by multiplying the center-of-mass Hamiltonian H_{cm} with an arbitrary factor (typically 10) and thereby shifting its first excited state to much higher excitation energies than the ones of the considered nuclear states [19].

2.2.3 Model space

To be able to calculate shell-model wave functions, the infinite number of single-particle states which contribute to the nuclear wave function has to be truncated in some way. This is usually achieved by only taking into account single-particle states within a certain range of excitation energies, characterized in terms of multiples of the harmonic-oscillator energy spacing $\hbar\omega$ (see figure 2.2). The residual interaction W_{ij} from equation 2.69 is then parameterized for a certain mass and excitation energy range in the form of an effective interaction, much like has been described for the effective NN interaction in section 2.1.3. The parameters of the effective interaction are adjusted to fit a broad range of experimentally determined level energies of the nuclei in question. In some cases the fitting procedure also includes experimental information about electromagnetic moments of the nuclei [40]. The most simple possible case is the limitation to 0 $\hbar\omega$ states. For the so-called p-shell nuclei, with masses $A = 4-16$, this means that only transitions within the $1p3/2$ and $1p1/2$ orbits are taken into account and the completely filled $1s$ shell is treated as an inert core. The best known parameterization

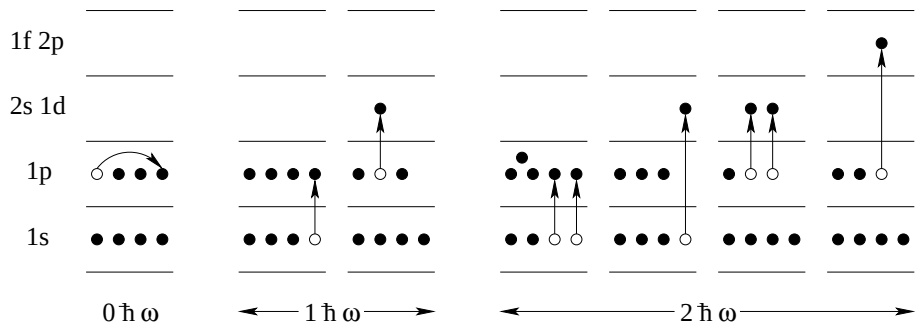

Figure 2.2: Schematic representation of 0 $\hbar\omega$, 1 $\hbar\omega$ and 2 $\hbar\omega$ model spaces in a mass $A = 7$ nucleus.

of the residual interaction for p-shell nuclei in a 0 $\hbar\omega$ model space has been performed by Cohen and Kurath [41].
To calculate non-normal-parity states, i.e. states for which the parity differs from the one of the ground state, it is necessary to extend the model space to include at least 1 $\hbar\omega$ excitations, which means in the case of p-shell nuclei to allow transitions from the $1s$ shell to the $1p$ shells and from the $1p$ to the $2s$ and $1d$ shells. Parameterizations of a $(0 + 1)$ $\hbar\omega$ interaction for p-shell nuclei have, for instance, been performed by Van Hees *et al.* [40].
The next possible extension to the model space is the inclusion of 2 $\hbar\omega$ excitations. For p-shell nuclei the configuration space then spans the $1s$, $1p$, $2s$, $1d$, $2p$ and $1f$ shells, allowing to perform complete $(0+2)$ $\hbar\omega$ calculations for normal-parity and restricted $(1 + 3)$ $\hbar\omega$ calculations for non-normal-parity states. The restriction in the latter case is that 3-$\hbar\omega$-type excitations can, in the named shells, only occur between the $1s$ and $2p$ or $1f$ shells. Parameterizations of the effective interaction for the above mentioned configuration space have among others been performed by Warburton and Brown [42] and Wolters *et al.* [43].
Numerical calculations of shell-model wave functions and related quantities in a certain model space can be performed using the code OXBASH [19]. The code comes with a large collection of effective interactions parameterized for different model spaces, some of which have been mentioned above.

2.3 Spin observables

The following sections present the formalism and the conventions used in the description of polarization-transfer experiments. The description of the

polarized cross section for reactions of type $\vec{\frac{1}{2}} + A \rightarrow \vec{\frac{1}{2}} + B$, where $\vec{\frac{1}{2}}$ represents polarized spin-$\frac{1}{2}$ particles, is deduced and some arguments about symmetries are given to limit the actual number of polarization observables. The presentation given here follows largely the review article by Ohlsen [44].

2.3.1 Spin density matrix ρ

Spin-$\frac{1}{2}$ particles are represented in quantum mechanics by Pauli spinors

$$\chi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} . \tag{2.77}$$

A physical observable is identified with the expectation value of a hermitian operator Ω which is given by

$$< \Omega >= \chi^{\dagger}\Omega\chi = (a_1^* \; a_2^*) \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^* & \Omega_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} . \tag{2.78}$$

Having an ensemble of N particles, the ensemble average of the operator Ω equals

$$\overline{< \Omega >} = \frac{1}{N} \sum_{n=1}^{N} \chi^{(n)\dagger}\Omega\chi^{(n)} \tag{2.79}$$

where the spinor $\chi^{(n)\dagger} = (a_1^{(n)}, a_2^{(n)})$ denotes the polarization state of the n^{th} spin-$\frac{1}{2}$ particle. Defining a density matrix

$$\rho = \frac{1}{N} \sum_{n=1}^{N} \chi^{(n)}\chi^{(n)\dagger} \tag{2.80}$$

equation 2.79 can be expressed as

$$\overline{< \Omega >} = Tr(\rho\Omega) . \tag{2.81}$$

Because when referring to an expectation value always the ensemble average is meant, the bar will be dropped in the following discussion.
The polarization $\vec{p}$ of an ensemble of N spin-$\frac{1}{2}$ particles is given by the expectation value of the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.82}$$

to be

$$\begin{aligned} p_x &\equiv < \sigma_x >= Tr(\rho\sigma_x) = \frac{1}{N}\sum_{n=1}^{N} 2Re(a_1^{(n)}a_2^{(n)*}) \ , \\ p_y &\equiv < \sigma_y >= Tr(\rho\sigma_y) = \frac{1}{N}\sum_{n=1}^{N} 2Im(a_1^{(n)}a_2^{(n)*}) \ , \\ p_z &\equiv < \sigma_z >= Tr(\rho\sigma_z) = \frac{1}{N}\sum_{n=1}^{N} (|a_1^{(n)}|^2 - |a_2^{(n)}|^2) \ . \end{aligned} \tag{2.83}$$

Values of the components of $\vec{p}$ lie in the range $-1 \leq p_i \leq +1$.
The expectation value of the unit matrix I gives the normalization of the density matrix

$$< I >= Tr(\rho) = \frac{1}{N}\sum_{n=1}^{N} (|a_1^{(n)}|^2 + |a_2^{(n)}|^2) = 1 \ . \tag{2.84}$$

Omitting the factor $\frac{1}{N}$ from the definition of the density matrix causes the expectation value $< I >$ to become equal to the number of particles N.
Because I, σ_x, σ_y and σ_z form a complete set of hermitian operators in 2×2 space the density matrix takes the form

$$\rho = \sum_{i=0}^{3} c_i\sigma_i \ , \tag{2.85}$$

where $(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \equiv (I, \sigma_x, \sigma_y, \sigma_z)$. Using

$$Tr(\sigma_i\sigma_j) = 2\delta_{ij} \ ; \ i, j = 0, 1, 2, 3 \ , \tag{2.86}$$

it follows that

$$\begin{aligned} p_j &\equiv < \sigma_j >= Tr(\rho\sigma_j) = Tr((\sum_{i=0}^{3} c_i\sigma_i)\sigma_j) \\ &= c_0 Tr(\sigma_0\sigma_j) + ... + c_3 Tr(\sigma_3\sigma_j) = 2c_j \end{aligned} \tag{2.87}$$

with $(p_1, p_2, p_3) \equiv (p_x, p_y, p_z)$ and $< \sigma_0 >=< I >= 1 = 2c_0$. This gives the result

$$\rho = \frac{1}{2}(I + \sum_{i=1}^{3} p_i\sigma_i) \ . \tag{2.88}$$

Equation 2.88 establishes a simple relation between the spin density matrix and the components of the polarization vector.

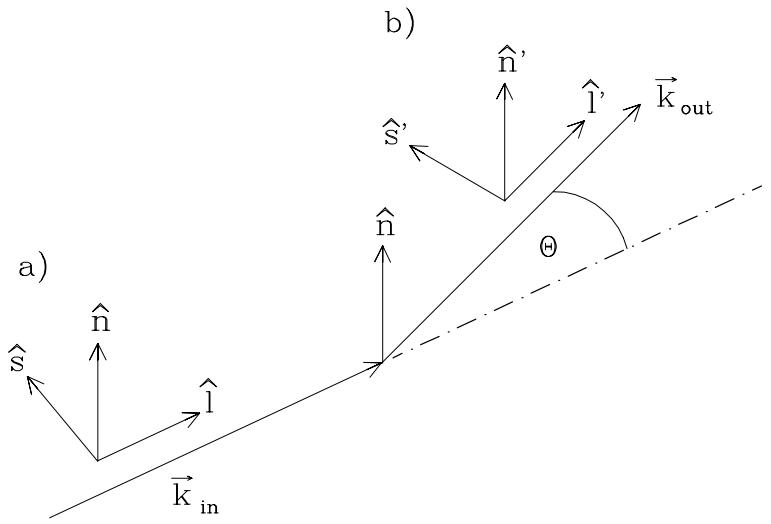

Figure 2.3: Coordinate systems used in the description of polarization transfer experiments: a) incoming helicity frame, b) outgoing helicity frame.

2.3.2 Coordinate systems

Polarization transfer experiments are generally described in so-called helicity coordinate systems. These are systems moving along with the particle, whose basis vectors $\hat{s}$, $\hat{n}$ and $\hat{l}$ (for sideways, normal and longitudinal) are given by the in- and outgoing momenta $\vec{k}_{in}$ and $\vec{k}_{out}$ of the projectile and ejectile (see figure 2.3). The incoming helicity frame is defined by

$$\hat{n} = \frac{\vec{k}_{in} \times \vec{k}_{out}}{|\vec{k}_{in} \times \vec{k}_{out}|} , \quad \hat{l} = \frac{\vec{k}_{in}}{|\vec{k}_{in}|} , \quad \hat{s} = \hat{n} \times \hat{l} \tag{2.89}$$

and the basis vectors of the outgoing helicity frame $\hat{s}\,'$, $\hat{n}\,'$ and $\hat{l}\,'$ are defined accordingly with

$$\hat{n}\,' = \hat{n} , \quad \hat{l}\,' = \frac{\vec{k}_{out}}{|\vec{k}_{out}|} , \quad \hat{s}\,' = \hat{n}\,' \times \hat{l}\,' . \tag{2.90}$$

2.3.3 Spin observables for the case $\frac{\vec{1}}{2} + A \rightarrow \frac{\vec{1}}{2} + B$

To simplify the derivation of the spin observables for the $\frac{\vec{1}}{2} + A \rightarrow \frac{\vec{1}}{2} + B$ reaction the following discussion will be limited to the case $\frac{\vec{1}}{2} + 0 \rightarrow \frac{\vec{1}}{2} + 0$, i.e. where the spins of the nuclei A and B are zero. All of the arguments will

remain valid for the reaction of interest and the general formalism has been described by Wolfenstein [45]. Conforming to the notation of Ohlsen [44] the polarized and unpolarized differential cross sections will in the following be labeled by $I(\theta,\phi)$ and $I_0(\theta)$, respectively.
The spinors of the projectile and ejectile are connected by the relation

$$\chi_f = M\chi_i \ , \tag{2.91}$$

with M being a 2×2 matrix, whose components depend on the bombarding energy, energy loss and scattering angle, and the subscripts i and f denote initial and final states of the particle. The density matrix of the final state

$$\rho_f = \frac{1}{N}\sum_{n=1}^{N}\chi_f^{(n)}\chi_f^{(n)\dagger} \tag{2.92}$$

follows from equation 2.91 and

$$\rho_i = \frac{1}{N}\sum_{n=1}^{N}\chi_i^{(n)}\chi_i^{(n)\dagger} \tag{2.93}$$

to be

$$\rho_f = M\rho_i M^\dagger \ . \tag{2.94}$$

If ρ_i is normalized to unity, the differential cross section for a polarized beam is given by

$$I(\theta,\phi) = Tr(\rho_f) = Tr(M\rho_i M^\dagger) \ . \tag{2.95}$$

For an unpolarized beam with

$$\rho_i = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{2.96}$$

the cross section reduces to

$$I_0(\theta) = \frac{1}{2}Tr(MM^\dagger) \ . \tag{2.97}$$

Combining equation 2.88 and 2.94 one obtains for the density matrix of a polarized beam

$$\rho_f = \frac{1}{2}MM^\dagger + \frac{1}{2}\sum_{i=1}^{3}p_i M\sigma_i M^\dagger \tag{2.98}$$

and therefore

$$I(\theta,\phi) = Tr(\rho_f) = I_0(\theta)(1+\sum_{i=1}^{3}p_i A_i(\theta)) \ , \tag{2.99}$$

with

$$A_i(\theta) \equiv \frac{Tr(M\sigma_i M^\dagger)}{Tr(MM^\dagger)} \tag{2.100}$$

being the analyzing power for the i^{th} component of the incoming polarization vector. The components of the outgoing polarization $p_{j'}$ are given by

$$p_{j'} \equiv < \sigma_{j'} > = \frac{Tr(\rho_f \sigma_{j'})}{Tr(\rho_f)} \ . \tag{2.101}$$

Applying equations 2.95, 2.97 and 2.98 one obtains the relation

$$p_{j'} I(\theta, \phi) = I_0(\theta)(P_{j'}(\theta) + \sum_{i=1}^{3} p_i D_{ij'}(\theta)) \ , \tag{2.102}$$

with

$$P_{j'}(\theta) = \frac{Tr(MM^\dagger \sigma_{j'})}{Tr(MM^\dagger)} \tag{2.103}$$

and

$$D_{ij'}(\theta) = \frac{Tr(M\sigma_i M^\dagger \sigma_{j'})}{Tr(MM^\dagger)} \ . \tag{2.104}$$

$P_{j'}$ is the j^{th} component of the induced polarization which would be obtained by the scattering of an unpolarized beam and $D_{ij'}$ is the polarization transfer coefficient connecting the i^{th} component with the j^{th} component of the incoming and outgoing polarization vectors, respectively. Values for $A_i(\theta)$ vary between 0 and 1, values for $P_{j'}$ and $D_{ij'}$ between $+1$ and -1.

2.3.4 Number of observables

Looking at equations 2.99 and 2.102 it seems that there are in total 15 spin observables (three analyzing powers A_i, three induced polarizations $P_{j'}$ and a 3×3 matrix of spin transfer coefficients $D_{ij'}$). Some of them, however, drop out if one considers that the strong interaction conserves parity.
The particle momenta $\vec{k}_{in}$ and $\vec{k}_{out}$ change sign under parity transformation causing the basis vectors of the in- and outgoing helicity frames to transform like

$$\hat{s} \rightarrow -\hat{s} \ , \ \hat{n} \rightarrow \hat{n} \ , \ \hat{l} \rightarrow -\hat{l} \ . \tag{2.105}$$

Looking at the scattering of a spin-$\frac{1}{2}$ particle under parity transformation, negative and positive sideways polarizations transform into negative and positive polarizations, respectively (see figure 2.4 a). The spin-transfer matrix element $D_{ss'}$ is the same for both cases and the reaction is therefore

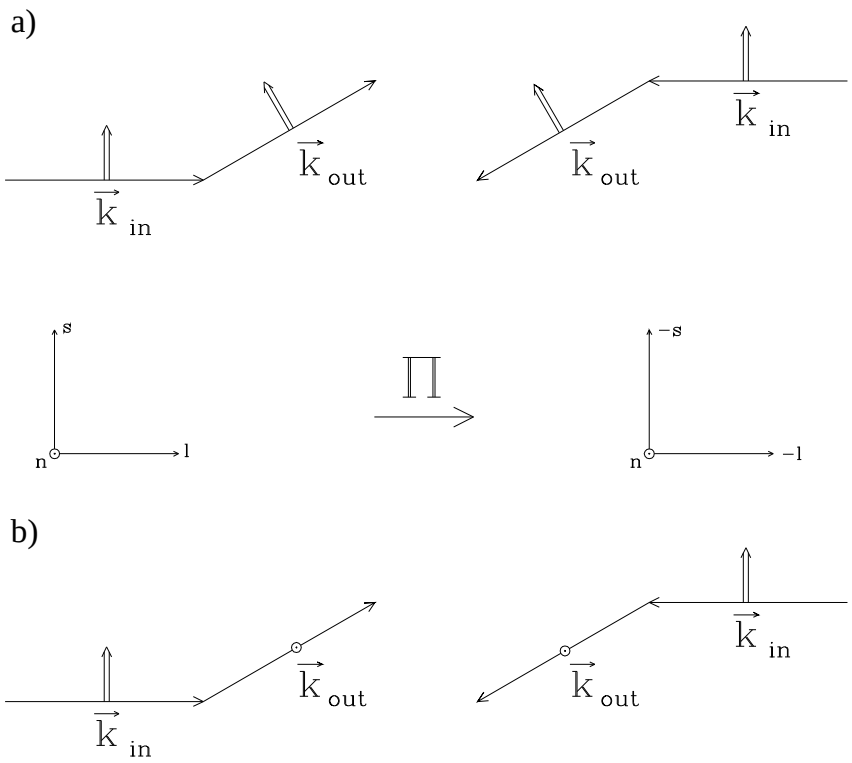

Figure 2.4: a) parity-allowed and b) parity-forbidden scattering of a spin-$\frac{1}{2}$ particle.

'parity allowed'.
In the case of the matrix element $D_{sn'}$ which transforms sideways into normal polarizations the component $+p_s$ goes to $+p_{n'}$ and $-p_s$ also goes to $+p_{n'}$ making the matrix element change sign under parity transformation. The reaction is therefore 'parity forbidden' (see figure 2.4 b).
Using analog arguments for the rest of the spin observables one finds that the following quantities have to be zero:

$$\begin{aligned} A_s = A_l = 0 \,, \\ P_s = P_l = 0 \,, \\ D_{sn'} = D_{ns'} = D_{nl'} = D_{ln'} = 0 \,, \end{aligned} \tag{2.106}$$

reducing equations 2.99 and 2.102 to

$$I(\theta, \phi) = I_0(\theta)(1 + p_n A_n(\theta)) \tag{2.107}$$

and

$$\begin{pmatrix} p_{s'} \\ p_{n'} \\ p_{l'} \end{pmatrix} I(\theta,\phi) = I_0(\theta) \left[\begin{pmatrix} 0 \\ P_{n'} \\ 0 \end{pmatrix} + \begin{pmatrix} D_{ss'} & 0 & D_{ls'} \\ 0 & D_{nn'} & 0 \\ D_{sl'} & 0 & D_{ll'} \end{pmatrix} \begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix} \right] . \tag{2.108}$$

It should be noted, that in the literature the analyzing power is mostly refered to as A_y instead of A_n, either because the helicity coordinate axes are labeled with $\hat{x}$, $\hat{y}$ and $\hat{z}$ instead of $\hat{s}$, $\hat{n}$ and $\hat{l}$ or because the scattering plane is fixed by the instrumental setup such that $\hat{n} = \hat{y}$, where $\hat{y}$ is defined in the laboratory system.
Applying invariance under rotation about the z axis one can derive whether a certain coefficient is an odd or an even function of θ. The result of this argument, which can be found in [44], is that the diagonal elements of the spin-transfer matrix $D_{ss'}$, $D_{nn'}$, $D_{ll'}$ are even and the rest of the coefficients A_n, $P_{n'}$, $D_{sl'}$ and $D_{ls'}$ are odd functions of θ. Specifically this implies that the analyzing power A_n and induced polarization $P_{n'}$ go to zero for $\theta \to 0$. Finally, it can be shown that for elastic scattering, which is invariant under time reversal, one has the following equalities:

$$A_n = P_{n'} \ , \ D_{sl'} = -D_{ls'} \ . \tag{2.109}$$

An important quantity for polarization transfer experiments is the transverse spin-flip probability $S_{nn'}$ which is defined by

$$S_{nn'} = \frac{1}{2}(1 - D_{nn'}) \ . \tag{2.110}$$

It ranges from 0 to 1 and is a direct measure of the spin-flip contribution to the cross section.